1. Give an example of a second order linear homogenous recurrence relation with constant coefficients.

Ans:

Problem 1: Second-Order Linear Homogeneous Recurrence Relations with Constant Coefficients – Distinct Roots.
Suppose a sequence satisfies the below given recurrence relation and initial conditions. Find an explicit formula for the sequence.

\[ a_k = 2a_{k-1} + 3a_{k-2} \text{ for all integers } k \geq 2 \]
\[ a_0 = 1, \ a_1 = 2 \]

Problem 2: Second-Order Linear Homogeneous Recurrence Relations with Constant Coefficients – Single Root.
Suppose a sequence satisfies the below given recurrence relation and initial conditions. Find an explicit formula for the sequence.

\[ n_k = 2n_{k-1} - n_{k-2} \text{ for all integers } k \geq 2 \]
\[ n_0 = 1, \ n_1 = 4 \]

Example: \( f(1)=1 \)
\( f(2)=2 \)
\( f(n)=4f(n/2) - 4f(n/4), \ n>2 \)

Change variables \( n \rightarrow 2^k \) (noting that as \( n/2 \rightarrow 2^{k-1}, n/4 \rightarrow 2^{k-2} \)) to obtain the recurrence for the new variable \( k \)

\[ f_k = 4f_{k-1} - 4f_{k-2} \]

This is a homogeneous 2\textsuperscript{nd} order recurrence, and should be solved using the characteristic equation method.

The characteristic equation for the above recurrence (variable \( k \)) is

\[ \alpha^2 - 4\alpha + 4 = 0 \]
\[ \rightarrow (\alpha - 2)^2 = 0 \]

Repeated root \( \rho=2 \), so the general solution for \( k \) is

\[ f_k = c_1 2^k + c_2 k 2^k \]

In terms of the original variable \( n \), this general solution is

\[ f(n) = c_1 n + c_2 n \log_2 n \]

(a): Find the order and degree of the following recurrences relation. Which of the following belongs to the linear homogenous recurrence relation with constant coefficient?

(i) \( f_n = f_{n+1} + f_{n-2} \)
(ii) \( a_n = 5a_{n+1} + n^3 \)
(iii) \( a_n = a_{n-1} + a_{n-2} + \ldots + a_0 \)
(iv) \( a_n = 5a_{n-1} - a_{n-2} \)

Ans: